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Title:	Gaussian process regression for radiological contamination mapping
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Gaussian process regression for radiological contamination mapping

Applied to optimal motion planning for mobile sensor platforms

Map contamination faster with autonomous controls

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Radiological contamination mapping

Applications in emergency response, tactical mission planning, and post-event procedure development, among others

What makes a good survey of the area?

1. Spatial resolution of mapped area
2. Good spatial coverage of the area
3. Identification of multiple contaminated areas (typically planar sources)

Want to achieve best representative characterization of the entire area **efficiently** and **accurately**

Unmanned aerial/ground vehicles (UAV/UGVs) for contamination mapping

UAV/drone systems

Sensors on board

Human operator

Safety + efficiency

Major Challenges

LIMITED BATTERY LIFE

Move smart

HUMAN OPERATED

Fully autonomous controls

MANY MEASUREMENTS

Predictive mapping capabilities

Current methods use uniform survey routines

Typically a raster-type motion is used with a human operator

Can only characterize contamination in the vicinity of where the sensors have been

Objective: Develop fully autonomous controls for mobile sensor platforms to improve efficiency and maintain performance

Many measurements → predictive mapping capabilities with sparse observations

Limited battery life → move sensors in optimal trajectories

Human operation → fully autonomous motion planning procedure

Our approach:

1. Gaussian process regression technique for full-map predictions
2. Voronoi partitions for maximized areal coverage
3. Recursive procedure for optimal motion trajectories for sensors

Gaussian Process Regression (GPR) model (1/5)

GPR allows us to predict spatial characteristics by using previously observed data

We assume the data can be modeled as a Gaussian process (GP), defined as **a collection of random variables where any finite number of the random variables have a joint Gaussian distribution**

$$\boldsymbol{y} = f(\boldsymbol{x}) \sim GP\big((m(\boldsymbol{x}), k(\boldsymbol{x}, \boldsymbol{x}')\big)$$

With mean function m(x) and covariance function k(x, x') defined as

$$m(\boldsymbol{x}) = \mathbb{E}[f(\boldsymbol{x})]$$

$$k(\boldsymbol{x}, \boldsymbol{x}') = \mathbb{E}[f(\boldsymbol{x}) - m(\boldsymbol{x}))(f(\boldsymbol{x}') - m(\boldsymbol{x}'))]$$

Gaussian Process Regression (GPR) model (2/5)

For our application, the training data represented as

$$(\mathbf{X} = (\mathbf{x_1},\ldots,\mathbf{x_n})^T, \mathbf{y} = (y_1,\ldots,y_n))$$

X contains features of the output data (n x 2 matrix with latitude and longitude)

y contains output of the data (n x 1 vector with radiation count rates)

Full GPR model defined as

$$\begin{pmatrix} y_* \\ y_1 \\ \vdots \\ y_n \end{pmatrix} \sim \mathcal{N}\left(\mu = \begin{pmatrix} m(\boldsymbol{x}_*) \\ m(\boldsymbol{x_1}) \\ \vdots \\ m(\boldsymbol{x_n}) \end{pmatrix}, \Sigma = \begin{pmatrix} k_{**} & k_{*.} \\ k_{.*} & k_{..} \end{pmatrix} \right)$$

y_* is the radiation count rates we are predicting at $\mathbf{x}_*$ unvisited locations

Gaussian Process Regression (GPR) model (3/5)

Covariance function has the form

$$k(\boldsymbol{x}_i, \boldsymbol{x}_j) = \sigma^2 R(|\boldsymbol{x}_i - \boldsymbol{x}_j|) = \sigma^2 R(\boldsymbol{r}),$$

$\boldsymbol{\sigma^2}$ is the variance of the GP (e.g., statistical uncertainty), **r** is the Euclidean distance between two feature sets, and **R(r)** is the correlation kernel to describe shape of covariance

The correlation kernel describes how $y(\mathbf{x}_i)$ and $y(\mathbf{x}_j)$ are related based on the similarity of the input feature sets $\mathbf{x}_i$ and $\mathbf{x}_j$ (i.e., Euclidean distance between locations).

Gaussian Process Regression (GPR) model (4/5)

K-fold cross-validation method to verify fidelity of correlation kernel in learning process

1. Take k number of partitions from fixed data set
2. Use the k^{th} partition as training set, and remainder of data as test set
3. Calculate metrics to evaluate "score" of predicting test set

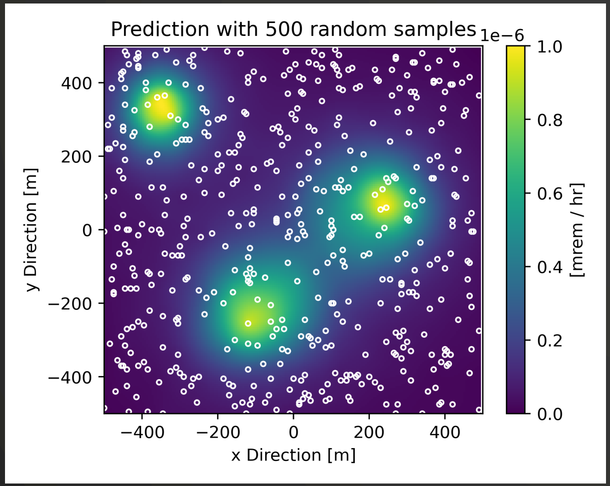
Dataset of 5000 samples used with 5-fold cross validation

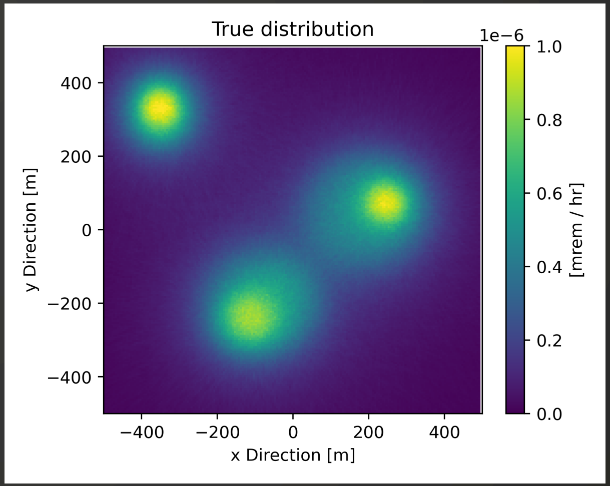
Kernel	RBF	RQ	Matern, v = 1/2	Matern, v = 3/2	Matern, v = 5/2
Fold1	0.118128	0.057216	0.051642	0.060256	0.069187
Fold2	0.133275	0.057208	0.051003	0.061374	0.071352
Fold3	0.130518	0.05333	0.048089	0.059527	0.070023
Fold4	0.121312	0.055755	0.050824	0.061835	0.072062
Fold5	0.141364	0.055168	0.051861	0.061844	0.07426
Average	0.128919	0.055735	**0.050684**	0.060967	0.071377

Matérn covariance with v = ½ gives best results

Gaussian Process Regression (GPR) model (5/5)

- MCNP 6.2 simulations of three large planar sources
- 500 random samples (out of 40,000) used to for full-map prediction
- Depends strongly on the location of the samples, does not produce distribution for samples < 500





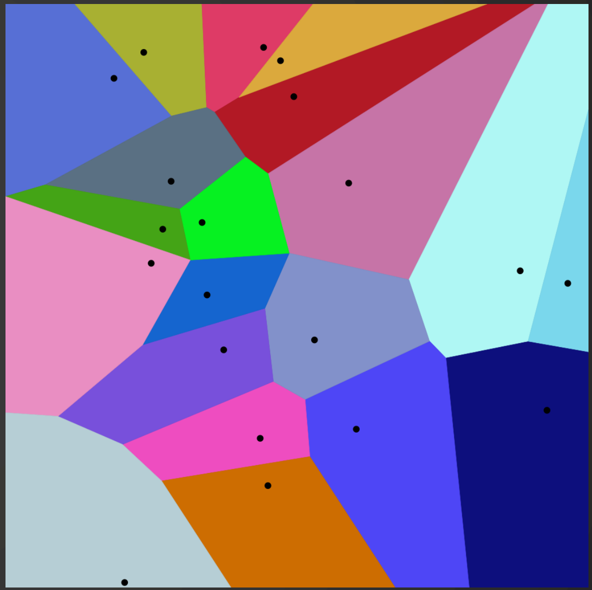
Voronoi Partition for optimal coverage (1/3)

Voronoi diagram is a partition of a plane into regions close to each of a given set of seed

For each seed, the corresponding Voronoi cell consists of all points of the plane closer to that seed than any other

Seeds = sensors

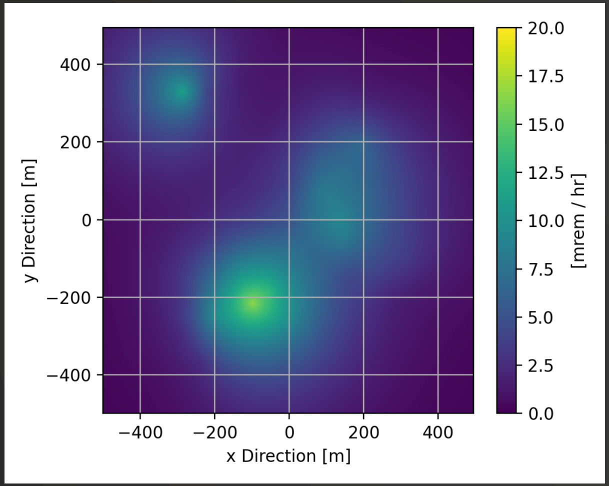
$$V_n(S) = \{g \in G \mid ||g - s_n|| \leq ||g - s_m||,\ for\ all\ s_m \in S, for\ n\ sensors$$



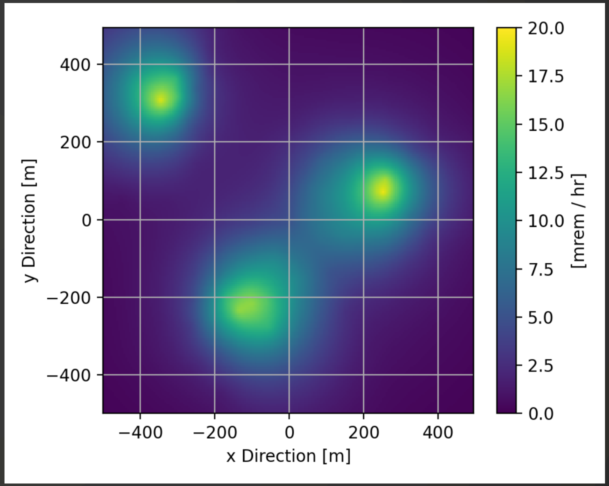
Voronoi Partition for optimal coverage (2/3)

How to get actionable information from GPR predicted map?

Take the difference between our current and prior prediction



Prior prediction $\phi_{j-1}(g)$

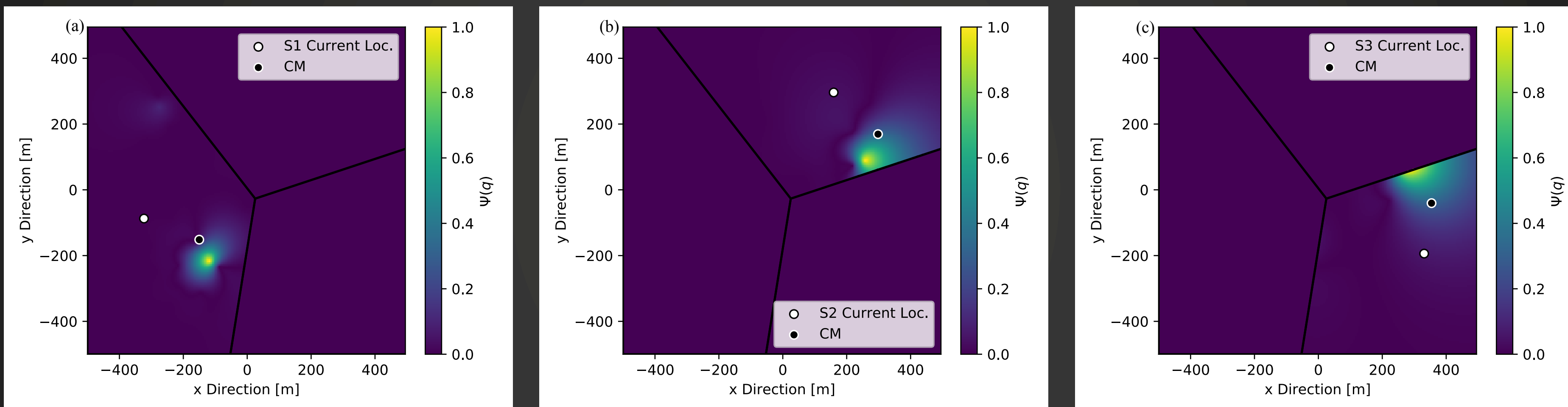


Current prediction $\phi_j(g)$

$$\Psi(g) = \left|\phi_j(g) - \phi_{j-1}(g)\right|$$

Voronoi Partition for optimal coverage (3/3)

Difference of prior and current map used as spatial density function for each Voronoi cell



Voronoi cells for a three-sensor survey showing current location (white circle), absolute difference between prior and current predictions, and the calculated trajectory (towards black circle)

Recursive motion planning procedure in Python (1/1)

Initialize:
1. Number of sensors
2. Sensor starting location
3. Uniform prior map

Take measurement and predict map Convergent map?

No

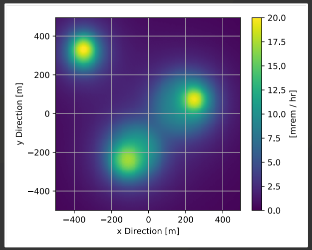
Calculate difference of map and Voronoi cells

Move sensors towards optimal direction

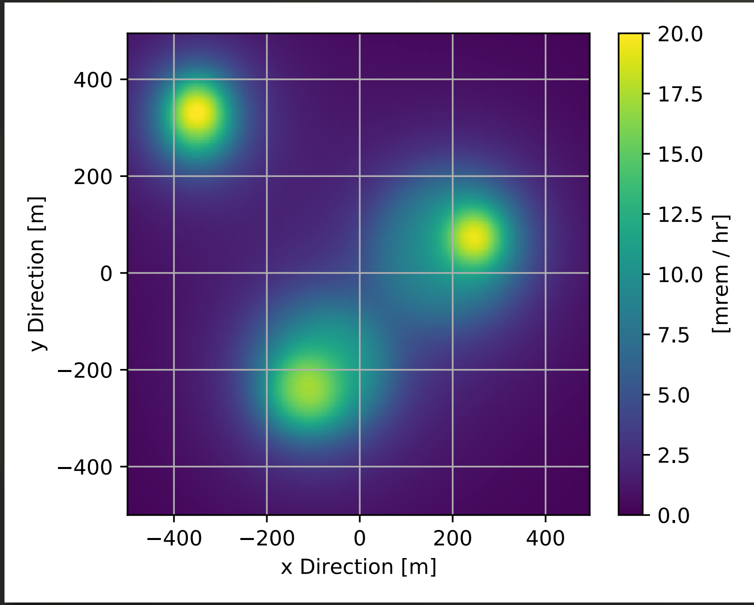
Yes

Calculate full-map prediction uncertainties

Final predicted contamination map



MCNP 6.2 simulations of radiological contamination (1/1)



$$E = 1 - \frac{\sum_{g \in G} |\phi_{true}(g) - \phi_{pred}(g)|}{\sum_{g \in G} \phi_{true}(g)}$$

Three large planar sources with varying activity of Cs-137 gamma rays, scaled to 10^{15} Bq

30 cm soil at ground level

1 km^2 area with 40,000 grid points

(5 x 5 x 5 m binning)

Sensor velocity = 5 m/s

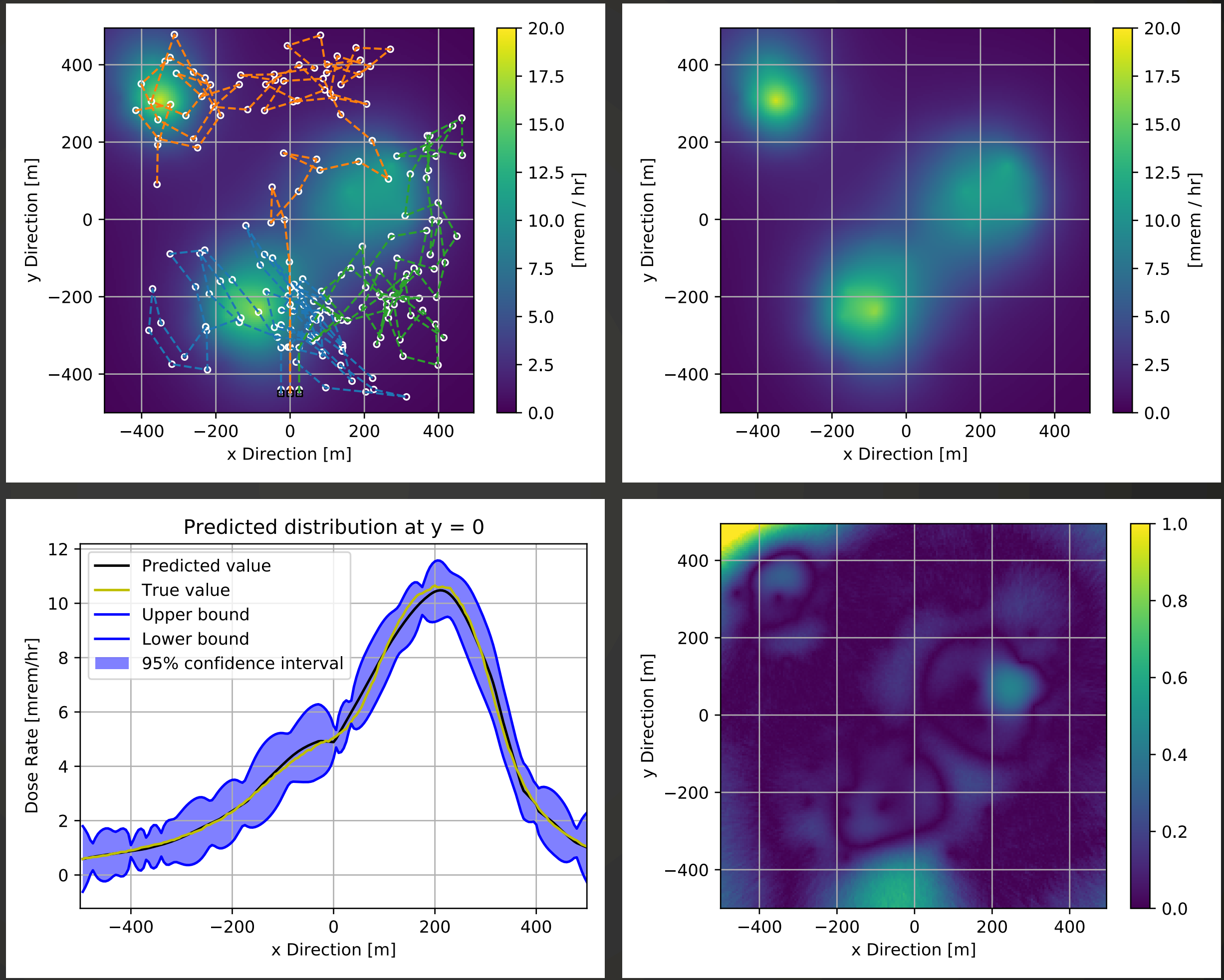
Dwell time for each measurement= 5 s

Ideal detector efficiency (100%)

Results (1/5) Three-sensor GPR survey

For an E = 0.90, three-sensor GPR routine takes approximately 1184 s total survey time

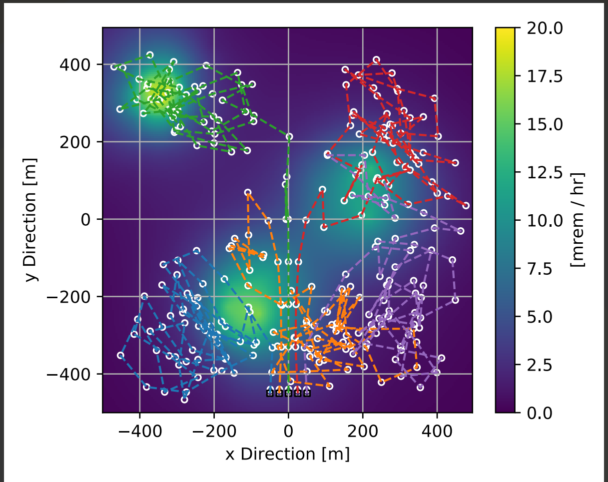
Prediction uncertainty dependent on proximity of observed measurement locations

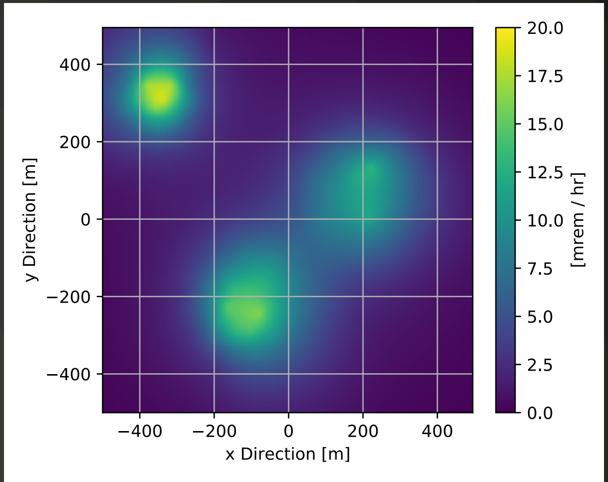


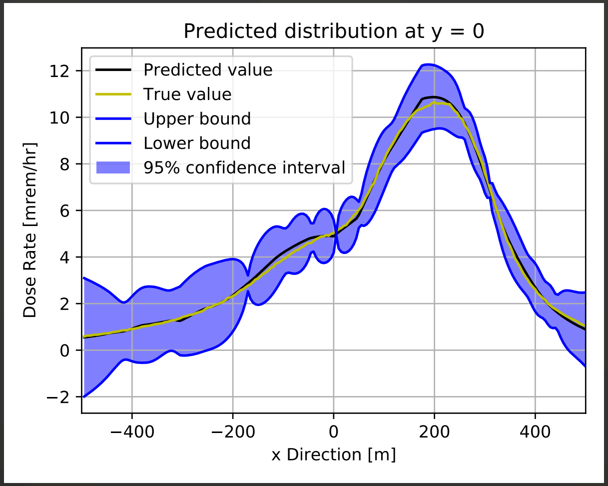
Results (2/5) Five-sensor GPR survey

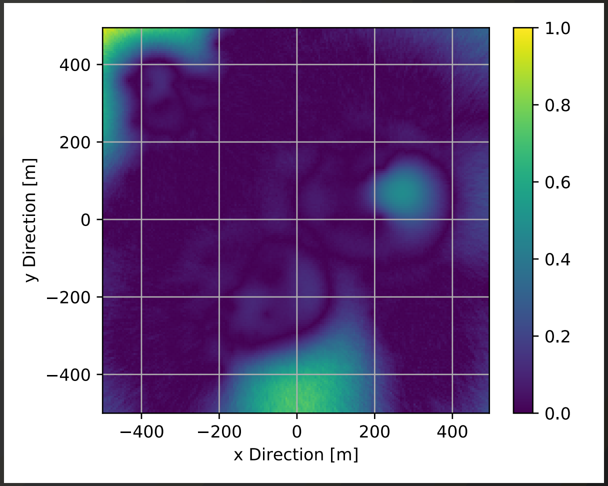
For an E = 0.90, five-sensor GPR routine takes approximately 745 s total survey time

Prediction uncertainty dependent on proximity of observed measurement locations





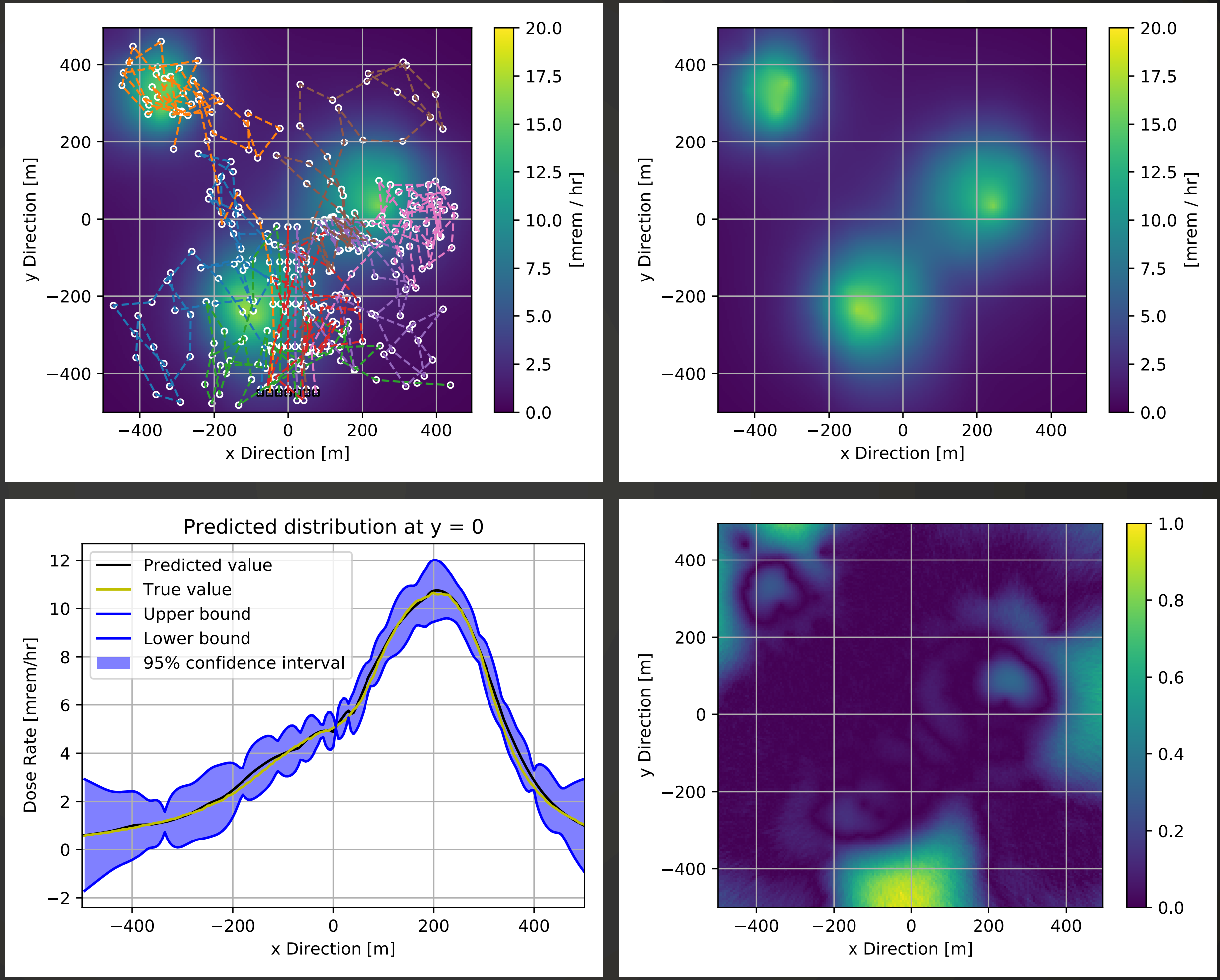




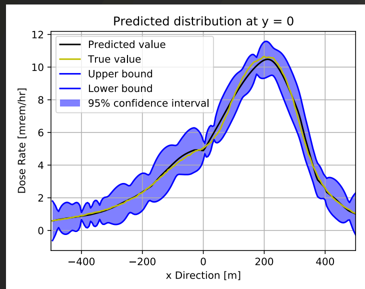
Results (3/5) Seven-sensor GPR survey

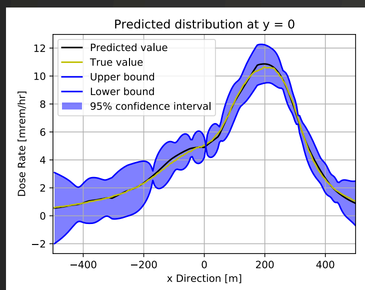
For an E = 0.90, seven-sensor GPR routine takes approximately 594 s total survey time

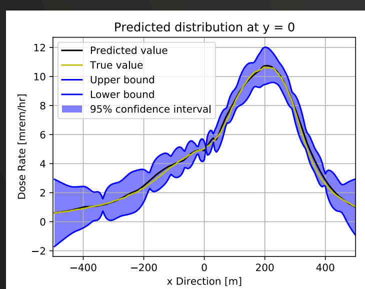
Prediction uncertainty dependent on proximity of observed measurement locations



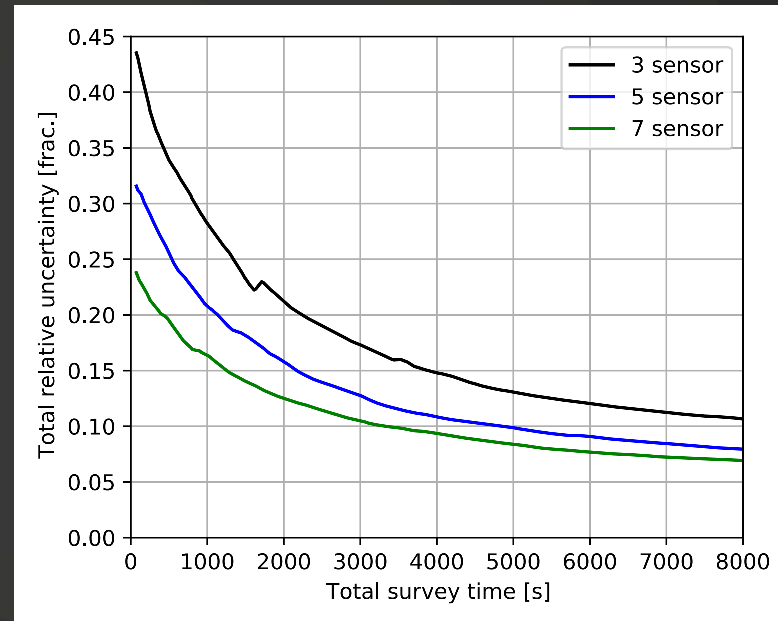
Results (4/5)
Long-term prediction uncertainty





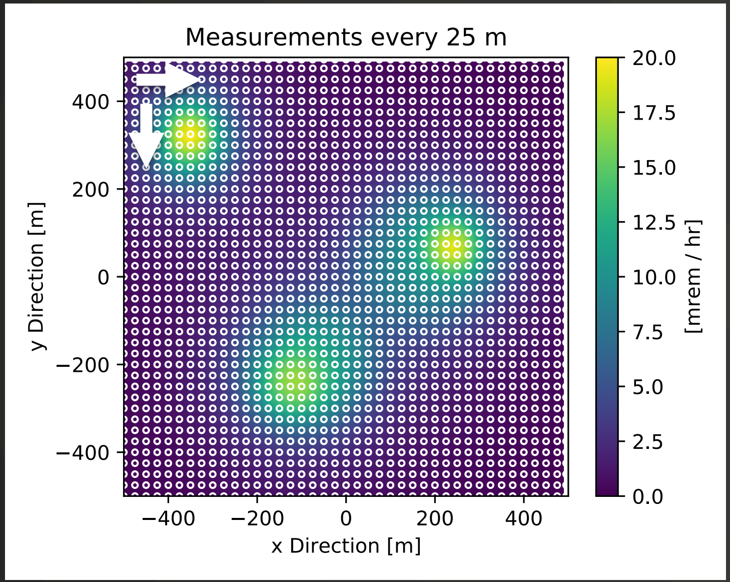


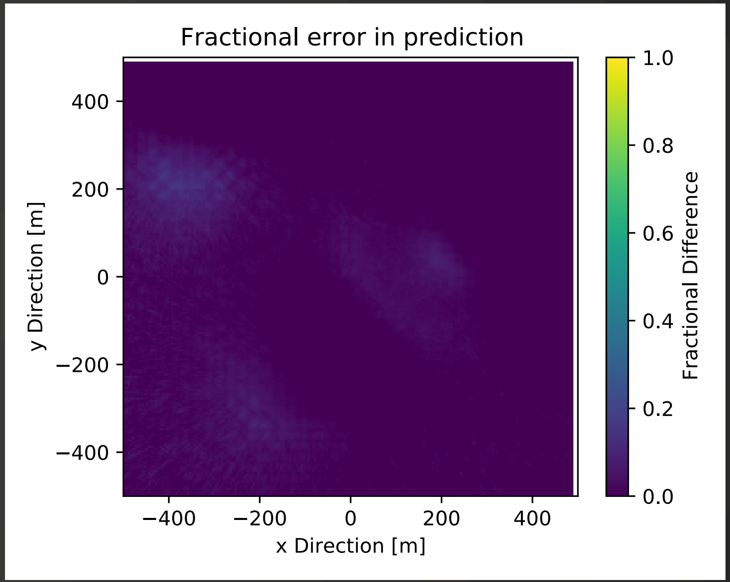
Average for entire area



Total prediction uncertainty decreases as more measurements are taken

Comparison to uniform survey routine





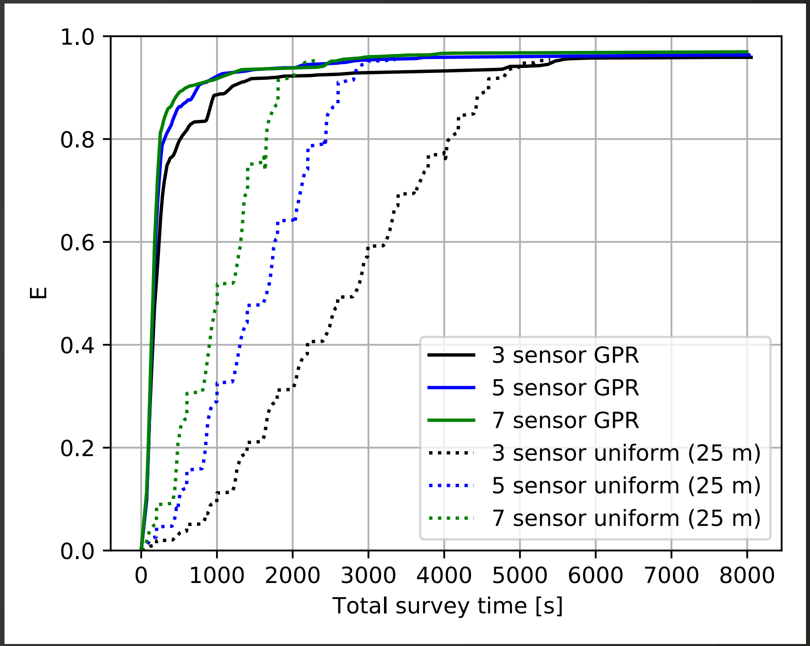
Uniform survey routine in raster-type motion, top to down, left to right.

Results (5/5)
Comparison to uniform survey routine

GPR survey routine converges to the same accuracy given by uniform routine

GPR survey routine produces convergent map at a faster rate

Increase in speed of producing map while maintaining accuracy



Summary and Conclusions

Developed a fully autonomous motion planning procedure using GPR predictive mapping and Voronoi-partition-based optimal coverage techniques

Simulation studies show that GPR-based survey routine produces an accurate map at a faster rate compared to uniform routine

GPR-based survey also provides prediction uncertainties beyond that of counting statistics → can provide quantitative metric to determine stopping criteria for surveys

Some other studies/findings

The GPR-based routine requires *at least* two mobile sensors; however, it is scalable to any number of sensors

Inherently has collision avoidance amongst the sensors due to the Voronoi partitioning

Efficacy of GPR-based routine is independent of number of sources and their relative source strength

Performs well with uniform background rates (tested with uniform background rate 1/100 of maximum source strength)

Moving forward… let's test it

Three programmable UGVs purchased

Features:
- 80 lbs payload
- ~ 2.5 m/s velocity
- Negligible turning radius (can turn in place)
- Continuously updatable waypoints

On-board Hardware:
- Wireless router for data and controls
- Arduino microcontroller
- GARMIN GPS and IMU

Simple Geiger counters + Raspberry Pi

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Thank you

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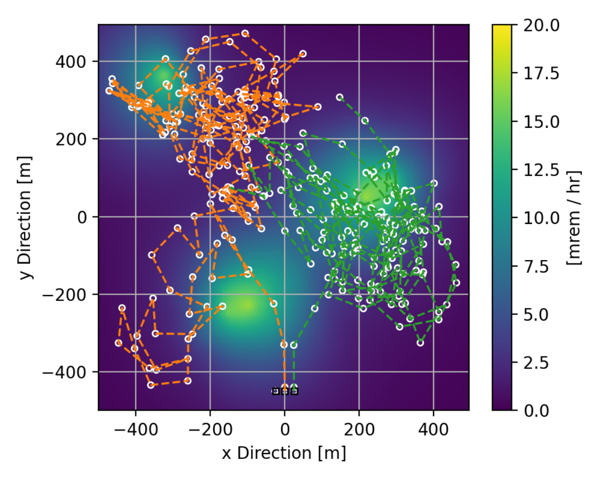
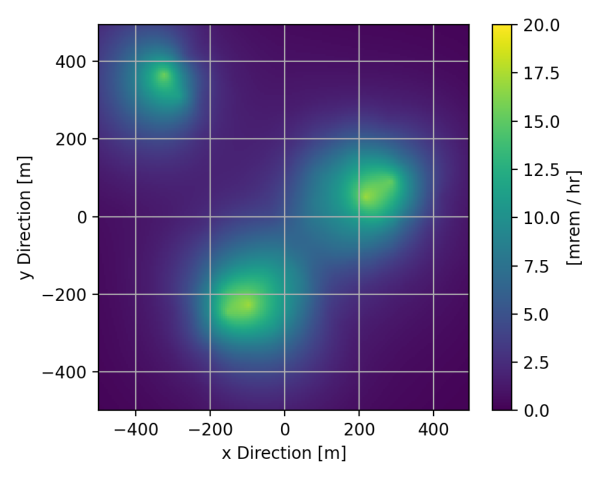
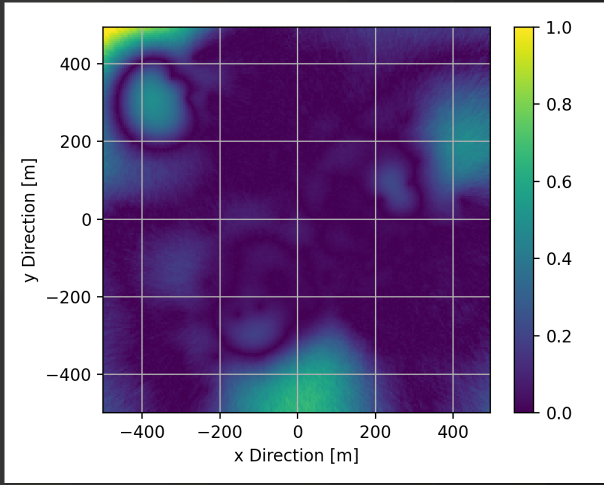
Extra slides: 2 moving sensors, 1 static sensor







Extra slides: seven-sensor with strong background